

Documentation of Formulas used in
Report of the American Academy of Actuaries'
Commissioners Standard Ordinary (CSO) Implications Work Group
as presented to the National Association of Insurance Commissioners'
Life and Health Actuarial Task Force

This documentation is for a Level Premium \$1,000 Whole Life policy.

Let: x = Issue Age;
 t = Duration;
 i = Interest Rate;
 d = $i / (1 + i)$;
 δ = $\ln(1 + i)$;
 FC = Fully Continuous Functions;
 SC = Semi-Continuous Functions; and
 R = Results shown in Report;

Using standard commutation functions notation, let:

$$\begin{aligned}\bar{A}_x &= \bar{M}_x / D_x ; \\ \bar{a}_x &= \bar{N}_x / D_x ; \text{ and} \\ \ddot{a}_x &= N_x / D_x .\end{aligned}$$

Let the following reflect Net Annual Premium and Terminal Reserve calculations:

$$\begin{aligned}\alpha^{FC} &= 1,000 * (\bar{C}_x / D_x) / (\bar{D}_x / D_x) ; \\ \beta^{FC} &= \bar{P}(\bar{A}_{x+1}) = 1,000 * \bar{A}_{x+1} / \bar{a}_{x+1} ; \text{ and} \\ {}_tV_x^{FC} &= 1,000 * \bar{A}_{x+t} - \beta^{FC} * \bar{a}_{x+t}, \quad t \geq 1 .\end{aligned}$$

The following are the formulas used for the Net Annual Premiums and Mean Reserves shown in the Report:

$$\begin{aligned}\alpha^R &= \alpha^{FC} * (d / \delta) ; & \beta^R &= \beta^{FC} * (d / \delta) ; \text{ and} \\ {}_tMV_x^R &= ({}_tV_x^{FC} + {}_tV_x^{FC} + \beta^R) / 2, \quad t \geq 1 .\end{aligned}$$

Let the following reflect Nonforfeiture Factor calculations:

$$\begin{aligned}NL^{SC} &= P(\bar{A}_x) = 1,000 * \bar{A}_x / \ddot{a}_x ; \\ E^{SC} &= (1.25 * NL^{SC}) + (0.01 * 1,000) ; \text{ and} \\ NFF^{SC} &= P^A(\bar{A}_x) = (1,000 * \bar{A}_x + E^{SC}) / \ddot{a}_x .\end{aligned}$$

The following is the formula used for the Cash Values shown in the Report:

$${}_tCV_x^R = 1,000 * \bar{A}_{x+t} - NFF^{SC} * \bar{a}_{x+t} .$$